AQA

AS
Further Mathematics

Statistics
Mark scheme

Specimen

Version 1.1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

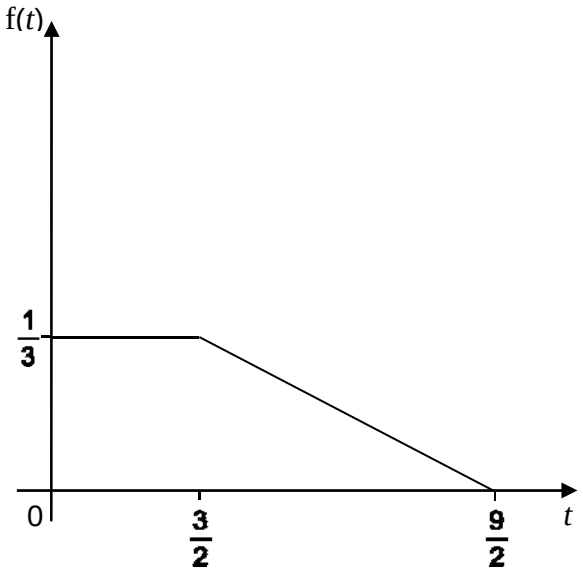
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	4
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	0.4
	Total		**1**	
3	Uses sum of probs = 1	AO1.2	M1	$0.4 + b + c = 1$ $b + c = 0.6$
	Uses formula for $\mathrm{E}(R)$	AO1.1a	M1	$\mathrm{E}(R) = 0.2 \Rightarrow$ $(-2\times0.3) + (0\times b) + (a\times c) + (4\times0.1) = 0.2$ $ac = 0.4$
	Uses formula for variance $\mathrm{E}(X^2) - (\mathrm{E}(X))^2$	AO1.1a	M1	$\mathrm{E}(X^2) - (\mathrm{E}(X))^2 = (4\times0.3) + (0\times b) + (a^2\times c) + (16\times0.1) - (0.2)^2 = 3.56$ $a^2c = 0.8$
	Obtains a, b and c CAO	AO1.1b	A1	From (2) and (3) $a = 2$ Hence $c = 0.2$ and $b = 0.4$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Uses Poisson λ =10 PI	AO1.1a	M1	$V + W$ is Po(10)
	Obtains correct probability	AO1.1b	A1	P(S >10) = 1 – P($S \leq 10$) = 0.417
(b)	States that model requires independence of purchases from store to store.	AO3.5b	E1	Purchases of printers at *Verigood* are not independent of those at *Winnerprint*
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Obtains w from $\int_0^{15} wl \, \mathrm{d}l = 1$ Allow one error if method correct	AO1.1a	M1	$\int_0^{15} wl \, \mathrm{d}l = w\left[\frac{l^2}{2}\right]_0^{15}$ $= \frac{225w}{2} = 1$ Hence $w = \frac{2}{225}$ $\mathrm{E}(L^2) = \int_0^{15} l^2 \times \frac{2}{225} l \, \mathrm{d}l$ $= \frac{2}{225}\left[\frac{l^4}{4}\right]_0^{15} = \frac{225}{2}$ $\mathrm{E}(S) =$ $\frac{1}{15}(0\times 0 + 1\times 1 + 2\times 2 + \ldots 5\times 5)$ $= \frac{55}{15} = \frac{11}{3}$ $\mathrm{E}(T) = \mathrm{E}(L^2) + \frac{1}{2}\mathrm{E}(S)$ $= \frac{225}{2} + \frac{11}{6} = \frac{343}{3} = 114\frac{1}{3}$ **AG**
	Obtains $\mathrm{E}(L^2)$ by integrating $\int l^2 \times \mathrm{f}(l) \, \mathrm{d}l$ FT 'their' value for w	AO1.1a	M1	
	Obtains $\mathrm{E}(S)$ by evaluating $\frac{1}{15}(0+1+2+3+4+5)$ Allow one error if method correct	AO1.1a	M1	
	Uses $\mathrm{E}(T) = \mathrm{E}(L^2) + \frac{1}{2}\mathrm{E}(S)$ (PI)	AO1.1a	M1	
	Shows that $\mathrm{E}(T) = 114\frac{1}{3}$ Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips **AG**	AO2.1	R1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6 (a)(i)	Draws horizontal line $(0, \frac{1}{3})$ to $(\frac{3}{2}, \frac{1}{3})$ and straight line joining $(\frac{3}{2}, \frac{1}{3})$ to t-axis	AO1.1a	M1	
	Sketches correct shape, accurate and fully labelled.	AO1.1b	A1	
(a)(ii)	Deduces median is $\frac{3}{2}$	AO2.2a	B1	Area under $f(t)$ for $0 \le t \le \frac{3}{2}$ is $\frac{1}{2}$ Median of t is $\frac{3}{2}$
(b)(i)	States integral required for E(T)	AO1.1a	M1	$\mathrm{E}(T) = \int_0^{\frac{3}{2}} \frac{1}{3} t \, \mathrm{d}t + \int_{\frac{3}{2}}^{\frac{9}{2}} \frac{t(9-2t)}{18} \mathrm{d}t$
	Obtains E(T) correctly	AO1.1b	A1	$= \left[\frac{1}{6}t^2\right]_0^{\frac{3}{2}} + \left[\frac{t^2}{4} - \frac{t^3}{27}\right]_{\frac{3}{2}}^{\frac{9}{2}}$ $= \frac{13}{8}$

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)(ii)	Uses definition for variance (PI)	AO1.1a	M1	$\text{Var}(T) = \text{E}\left(T^2\right) - \left(\text{E}\left(T\right)\right)^2$
	Obtains Var(T)	AO1.1b	A1	$\text{Var}(T) = \frac{15}{4} - \left(\frac{13}{8}\right)^2 = \frac{71}{64}$
	Obtains Var (4 T – 5) (Follow through 'their' Var (T))	AO1.1b	A1F	$\text{Var}(4T - 5) = 16\,\text{Var}(T) = \frac{71}{4}$
	Total		**8**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="2">7(a)</td><td>Obtains one missing expected value eg
Low and Breed A = $\frac{21\times 27}{105}$</td><td>AO1.1a</td><td>M1</td><td rowspan="2"><table><tr><th></th><th>Low</th><th>Med</th><th>High</th></tr><tr><td>A</td><td>5.4</td><td>9.6</td><td>6</td></tr><tr><td>B</td><td>5.14</td><td>9.14</td><td>5.71</td></tr><tr><td>C</td><td>8.23</td><td>14.63</td><td>9.14</td></tr><tr><td>D</td><td>8.23</td><td>14.63</td><td>9.14</td></tr></table></td></tr>
<tr><td>Obtains all expected values.
A0 if integers</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td rowspan="5">(b)(i)</td><td>States both hypotheses using correct language</td><td>AO2.5</td><td>B1</td><td rowspan="5">H_0 Milk yield is independent of breed
H_1 Milk yield is not independent of breed 1 tail 1%

ts = $\sum\frac{(O-E)^2}{E}=19.4$

χ^2 cv for 6 df = 16.81
(p = 0.00275)

19.4 > 16.81
(0.00275 < 0.01)

Reject H_0 and conclude that there is evidence to suggest that milk yield is not independent of breed</td></tr>
<tr><td>States critical value (or p-value)</td><td>AO1.1b</td><td>B1</td></tr>
<tr><td>Evaluates the χ^2 test statistic by comparing the cv with the ts (or p-value with 0.01)</td><td>AO3.5a</td><td>R1</td></tr>
<tr><td>Infers H_0 rejected</td><td>AO2.2b</td><td>E1</td></tr>
<tr><td>Concludes correctly in context (conclusion should not be definite)</td><td>AO3.2a</td><td>E1</td></tr>
<tr><td rowspan="2">(b)(ii)</td><td>Considers $\frac{(O-E)^2}{E}$ or $(O-E)$ to identify largest sources of association
[Do not allow mark if no reference to why source selected]</td><td>AO2.4</td><td>E1</td><td>Largest sources of association
Breed A/High and Breed B/ Low.</td></tr>
<tr><td>Interprets main source(s) of association in context</td><td>AO3.2a</td><td>E1dep</td><td>Far more than expected Breed A cows observed to have high milk yield
or
Far more than expected Breed B cows observed to have low milk yield</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	States both hypotheses using correct language	AO2.5	B1	H_0 $\lambda = 14$ H_1 $\lambda < 14$ 1 tail 5 % Allow $\lambda = 3.5$ used
	Selects and uses Poisson model with $\lambda = 14$ to find P(total sales ≤ 8) or P(total sales < 8)	AO3.3	M1	T ~ Po(14)
	Obtains 0.062	AO1.1b	A1	P(Total sales ≤ 8) = 0.0.62
	Evaluates the Poisson model by comparing 'their' p-value with 0.05	AO3.5a	M1F	p-value =0.062 0.062 > 0.05
	Infers H_0 accepted	AO2.2b	E1	Accept H_0
	Concludes in context (not definite)	AO3.2a	E1	No significant evidence to suggest that Keith's mean number of properties sold per week in spring has reduced.
(b)	States meaning in context of Type II error	AO3.2a	E1	Type II error is to conclude that there is no significant evidence to suggest that Keith's mean number of properties sold per week in spring has reduced when, in fact, the mean number sold has reduced.

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Advantage clearly stated	AO1.2	E1	**Advantage** Less likely to reject H_0 when H_0 is, in fact, true or Less likely to make a Type I error
	Disadvantage clearly stated	AO1.2	E1	**Disadvantage** Less likely to accept H_1 when H_0 is, in fact, false or More likely to make a Type II error
	Total		**9**	
	TOTAL		**40**	